Predicting Motor and Generator Maximum Torque as a Function of Mass

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***Abstract*— There is currently no universally accepted way to predict the maximum input/output torque of electrical generators/motors as a function of the machine's mass. A new correlation is proposed for this relationship that collapses the data from machines with masses ranging over more than three orders of magnitude and torques ranging over five orders of magnitude. It is shown that, at least for machines operating below 3600 rpm, this relationship is only weakly dependent on other parameters of the machine design such as operating rpm, motor type (inductance vs. permanent magnet), operating voltage etc… Physical arguments are given to explain the correlation and its independence from other operating parameters. This new correlation allows a system designer to have a simple one variable equation to calculate the mass of a motor or generator given a desired torque capability. Because the correlation is so predictive, it can also be used as a metric to compare different machine designs. Families of machines that have slightly higher torques than predicted by the correlation typically are "better" designs.**

***Keywords*— Electric Machines, Generators, Motors, Power, Torque**

I. Introduction

The design of many systems involving electric motors or generators often involve tradeoffs between the capability of the machine (here defined as the maximum continuous input or output torque) and the mass or size of the machine. An example is a wind turbine where a larger more expensive generator allows a lower ratio gearbox that is more reliable. Another example is in electric vehicles where larger motors result in greater torque but increase the mass of the vehicle. A simple model that predicts an electrical machine's size as a function of the torque requirements is needed to optimize these systems. In this paper, a correlation is created to describe this relationship. This correlation is applicable to typical machines used in industrial environments including induction motors, permanent magnet motors, and generators. Industrial motors and generators operating at speeds under 3600 rpm represent the vast majority of electrical machines in use today and are the ones targeted with this correlation.

II. Background on Electric Machine Metrics

Many different metrics are used to discuss the performance of an electric machine as a function of size/weight. Most use specific power or specific torque but even the definitions of these vary. Cavagnino [1] defines specific torque in Nm/kg where the kg is "active weight" which is further defined as only the magnetic and electrical portions of the motor. Oswald [2] defines specific torque in Nm/m^3 with the volume defined as the cylindrical volume not counting shaft or fins outside stator, but counting end turn and end cap size. Hoang [3] uses specific torque in units of Nm/kg but does not describe the basis for weight. When the size or weight includes only a portion of the machine, the metric is useful mainly for motor designers and less so for motor users. As shown below, none of these metrics are both independent of machine mass and include the entire size or weight of the machine so the performance of machines of different sizes cannot be compared using these metrics in a way that is useful for motor users.

Another metric used that has a more uniform definition is air gap magnetic shear stress or gap shear stress for short. This is generally defined as tangential force at the airgap radius divided by the surface area of the rotor [3]:

$$\tau_g = \frac{F_{rd}}{A_r}\tau_g = \frac{F_{rd}}{A_r} \tag{1}$$

where τ_g is the gap shear stress, Frd is the tangential force at rotor diameter, and Ar is the rotor surface area. Gap shear stress is sometimes described as "being the product of electric loading and magnetic loading"[4] and typical values are 6kPa to 140kPa [5] depending on motor size and thermal solution [5]. This is corroborated by Patterson looking at smaller machines claiming 20kPa is state of the art [4] and Hodge looking at larger machines claiming 80-120kPa is possible [6]. These size and thermal dependencies limit the usefulness of this metric. Additionally, gap shear stress is further limited from a user's perspective since rotor surface area is an unknown and an unimportant parameter for the system designer. Gap shear stress can be related to other metrics. Mongeau [7] defines power density in units of MW/m^2 calculated as gap shear stress times gap speed, $\tau_g V_{rd}$, where the gap speed, V_{rd}, is the tangential velocity at the airgap. Power density could also be written as P_m/A_r where Pm is the motor power because

$$P_m = T_m\omega = F_{rd}V_{rd} \tag{2}$$

where Tm is the motor torque and ω is the rotational speed.

Mongeau claims that 3MW/m^2 is the state of the art power density for Permanent Magnet (PM) machines [8] while implying this works over a wide size & speed range.

However, data from his presentation showed a range of 0.3MW/m^2 to 3.2MW/m^2 [8]. Further, it is hard to get data on commercial motors since rotor surface area is typically not published. This difficulty of knowing surface area reduces the usefulness to a motor user. Gap shear stress will be used as part of a baseline for the physics justification for the correlation and metric proposed below.

Any good metric will have as few dependencies as possible and ideally be non-dimensional. Specific torque regardless of the definition is dependent on the size of the machine with larger machines having higher specific torque resulting in specific torque being a function of size. Further, specific torque is also a function of technology with higher performance machines having higher specific torques. This leads to the functional representation:

$$specific\ torque = f(technology, weight) \quad (3)$$

For specific power, speed is also a dependence making the functional representation:

$$specific\ power = f(tech., weight, speed) \quad (4)$$

Ideally there would be a metric of motor performance that is only a function of technology without any other dependencies.

III. Proposed Corelation

A. Physics Based Background and Justification

Depending on the type of motor, torque is either proportional to current or current squared. Induction motors have torque proportional to current squared[9]. Surface Mount Permanent Magnet (SPM) motors have torque proportional to the current[10]. Internal Permanent Magnet (IPM) motors have two torque terms one that is proportional to current and the other that is proportional to current squared[11]. For the purposes of this metric development, the important characteristic is that the torque goes up when the current goes up.

The maximum operating power rating of a motor is determined by the allowed temperature of the windings. If more temperature capability is available, the motor can take more current and therefore more power and its rating would be higher.

Motors in industrial environments typically have speeds related to historical requirements that align the motor speed with an actual speed of a grid connected induction motor. In 60 hz locations, these are just below the synchronous speeds of 3600 rpm, 1800 rpm, 1200 rpm, 900 rpm for pole counts of 2, 4, 6, 8 respectively. In Europe or Asia where the power is 50 hz, the speed are 17% slower. This means the vast majority of applications are 3600 rpm or slower because it is not possible to make a motor with less than 2 poles. With modern Variable Speed Drives (VSDs) there is no longer the requirement to keep to these speeds, but historical use of these machines makes this speed range by far the most common.

As motors get larger, the ratio of torque to size or weight will increase. This can be supported by looking at the gap shear stress equation where possible gap shear stress increases as a function of size [5]. Rearranging the gap shear stress Equation 1 yields:

$$\boldsymbol{\tau_g = \frac{T}{2\pi {r_r}^2 l_r}} \quad (5)$$

Where T is the motor torque, r_r is the rotor radii, and l_r is the length of the rotor. (Gap shear stress is basically specific torque based on the rotor volume).

If a motor’s rotor length doubles, the specific torque will double, but the mass will not double because the end turns, bearings, and end plates will not increase in size. This would point to motor volume increasing less than linearly with the torque requirement.

The same is true for increasing the diameter of a motor. The stator/rotor can be approximated as a solid cylinder for smaller motors and a thin ring for larger motors. Considering that when the radii (r_r) doubles, if the gap shear stress stays constant or increases the torque will increase by at least a factor of 4 while the weight will increase by 2 to 4 times depending on the solid or thin cylinder assumption. This means the motor volume increases less than linearly with the torque requirement. If gap shear stress were constant, this would indicate that the volume would increase more than the square root of torque but with increasing gap shear stress there is not a clear lower bound.

Considering either length changes or diameter changes it is reasonable to consider a relationship

$$\boldsymbol{MNT = \frac{T}{M^{\alpha}}} \quad (6)$$

Where M is the motor mass (a more easily obtained metric than volume), T is the motor torque, α is an exponent between one and two, and MNT is a fitting constant which we call Mass Normalized Torque.

An exponent of one would represent a motor with zero length end turns and a solid cylinder assumption, and constant gap shear stress. An exponent of two would represent a total end turn length equal to the stack length, a thin cylinder assumption, and constant gap shear stress. Most cases will likely fall between these two exponents.

B. Metric Development

To understand this effect with real life industrial motors, the weights along with their maximum continuous torque of hundreds of motors were plotted in Fig. 1. These were selected from water-cooled and high performance air-cooled motors that range from 200 Nm to 2,700,000 Nm shown on a log-log plot. [7] [12-16] Three curve fits are shown all using a 1.5 exponent with three values of ***MNT*** (0.01,0.05 and 0.1 $Nm/kg^{3/2}$ The use of a 1.5 exponent generally matches expectations based on first principles by the fact that it is between 1 and 2. It can be seen in Fig. 1 that the overall slope of the data matches this 1.5 exponent over many orders of magnitude in mass and torque. Within each group of motors, the slope matches the exponent fit even closer.

Included in these data are air-cooled and water-cooled motors of all different configuations. Some like air-cooled TEFC, do not require an external cooling system, and others including forced cooled air motor and water cooled motors require an external cooling system. These were not included in the weight estimate because they are often part of a larger system and the cooling system weight is small compared with the motor.

Using this approximation gives the opportunity to use Mass Normalized Torque for individual motors:

$$MNT = \frac{T}{M^{1.5}} \tag{7}$$

as a metric to compare performance. Motors with a larger

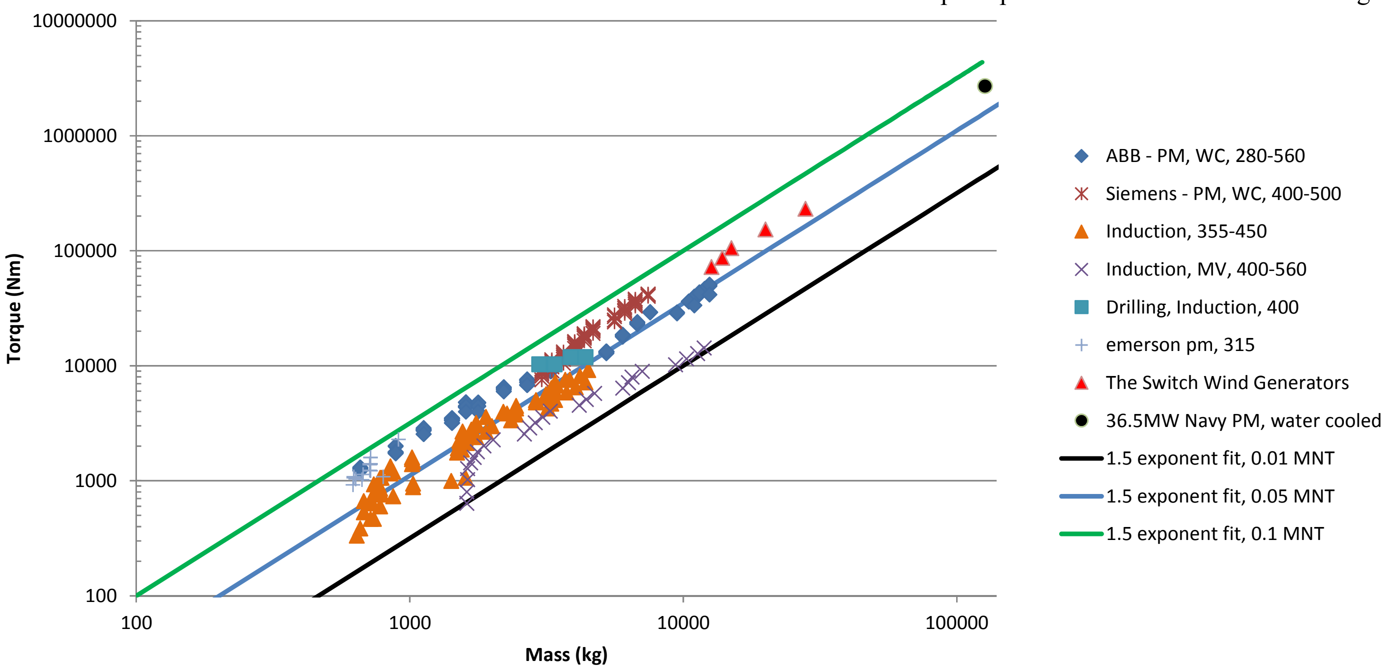

Fig. 1 – Torque vs Mass of Commercial Motors

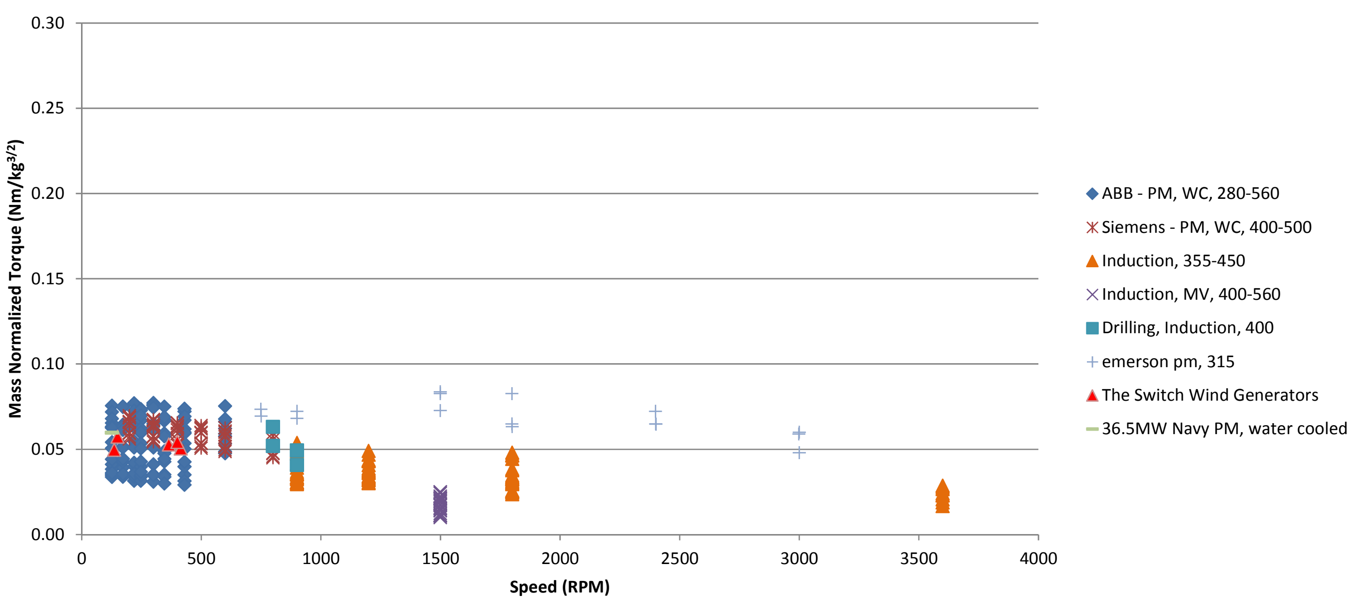

Fig. 2 – Mass Normalized Torque (MNT) vs speed on production motors and generators

value of MNT produce more torque for the same mass and thus are higher-performance designs. This metric is dimensional and requires using torque and mass in the proper units, but still provides a useful way to compare motors. The units of this proposed metric are $Nm/kg^{1.5}$.

Fig. 2 shows MNT versus speed for the same data as Fig. 1. Besides a slight reduction in MNT for machines over 1800 rpm where iron losses start to become a bigger factor, there is very little change in value with speed indicating that this metric is only a function of technology. If power were used rather than torque in the correlation, the data would not be independent of rpm. For example, for two machines with the same torque a 3600 rpm machine will have twice the power of an 1800 rpm machine since power = torque times rotation speed.

This MNT approach is different than other metrics that have been used to compare motors because it provides a way to compare different size and speed machines. One limitation is that it was only tested at up to 3600 rpm and is a better fit at less than 1800 rpm. However, this speed range encompasses the vast majority of all motors and generators used today. Another limitation is the fit data only included machines over 200Nm. No data was collected for smaller machines such as the fractional horsepower machines that are common in households today. To verify the low-torque predictions, the NEMA MG-1 2014 [17] US Standard for motors and generators is used. This standard gives frame sizes for TEFC class B motors for powers up to 250hp in multiple speed ranges. Fig. 3 shows the same data as Fig. 1 with the NEMA standard added (shown in open circles). Since the standard did not give the actual weights, it was necessary to correlate frame size to weight. It can be seen that the data is consistent for lower machine sizes. These NEMA machines are air-cooled induction machines and represent the lower side of performance thus have a slightly lower value of MNT.

Based on this data, most of the motors and generators have an MNT of between 0.03 and 0.07 regardless of speed. The air cooled induction motors shown with an X and the closed triangle have MNT values near 0.04 and are lower values than the water cooled permanent magnet machines. Air-cooled machines should have lower values of MNT due to the larger sizes needed with air-cooling. The air-cooled medium voltage machines shown with an X have the lowest values of MNT because these machines require thicker insulation.

Based on these results an electrical machine weight can be predicted by the equation 7. An MNT of 0.01 to 0.08 can be chosen based on the type of motor as shown in Table 1.

A water-cooled permanent magnet motor would be at the high part of this range and an air-cooled induction motor would be at the bottom part of the range. Equation 7, then gives an approximate weight of the machine given a specified torque capability.

TABLE 1 - TYPICAL MNT VALUES

	Range of MNT Values ($Nm/kg^{3/2}$)	Typical MNT Values ($Nm/kg^{3/2}$)
Industrial TEFC Induction Motors	0.01-0.03	0.02
High Performance Water Cooled Induction Motors	0.03-0.05	0.04
Industrial Water Cooled PM Motors	0.04-0.06	0.05
High Performance Water Cooled PM Motors	0.06-0.08	0.07

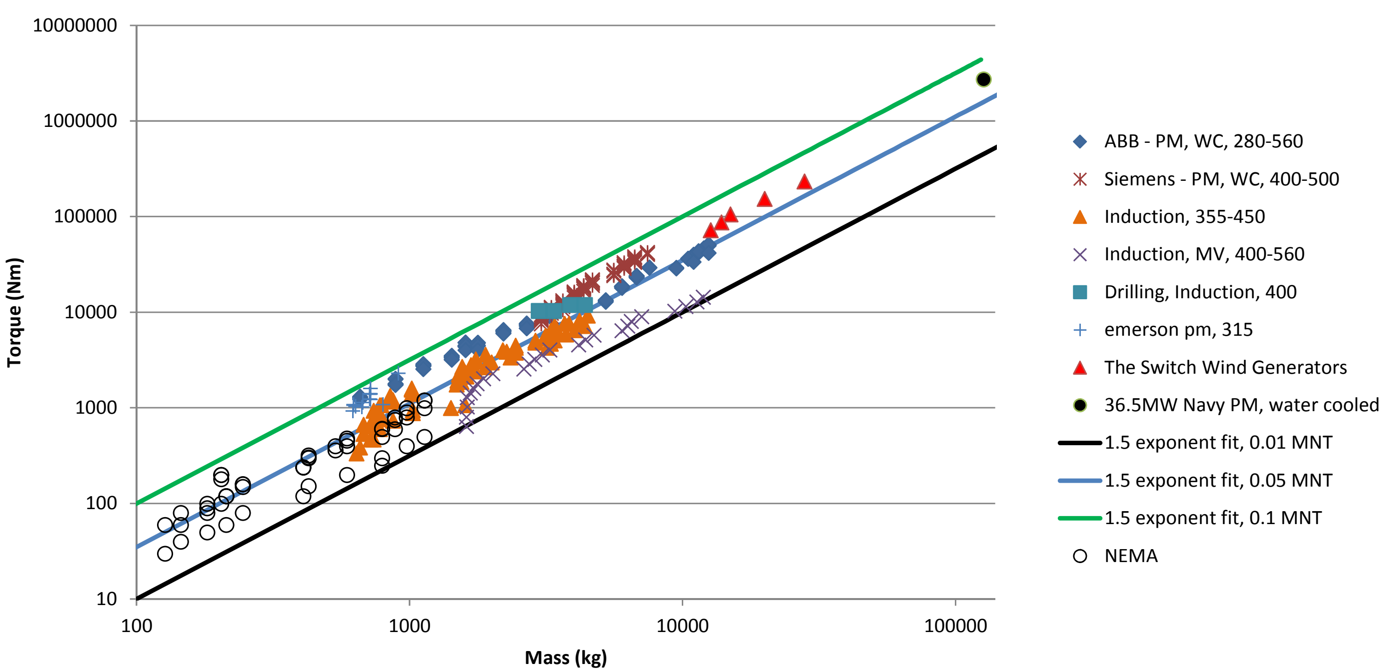

Fig. 3 - Torque vs Speed with NEMA motors added

IV. Examples

A. Motor Sizing

If one had an application for a marine propulsion motor that required 7500Nm at 800rpm (628kW), equation 7 could be used to estimate the possible weight of the motor. If weight is important, but an induction motor is desired, Table 1 could be used to estimate an MNT of 0.04. Using equation 7 and solving for M yields a mass of 3276kg. Searching for available water cooled induction motors for similar characteristics finds an ABB M3LP 450 LB 3GLP 454 520 motor that is 3700kg [12]. This is comparable to the predicted mass.

If smaller sizes are required, a water cooled PM motor could be chosen. This would allow an MNT of 0.05 per Table 1. Using equation 7 yields a motor weight of 2823kg. Inspecting available water cooled PM motors finds a Siemens 1FW4401-1HF motor that is 3040kg [14]. Again, this is somewhat close to the prediction considering the amount of information used.

If smaller sizes were still required it would require finding a more exotic solution, adding a gearbox, or other system solution to reduce the required size.

B. System Optimization

If one were designing a system such as a marine propulsion system, variables would include propeller diameter, pitch, and rpm as well as gearbox ratio all with achieving the same propulsion power. Each of these variables affects the torque required and therefore the size of the required motor. For a first order optimization of system weight, using equation 7 would allow the engineer to make a prediction of the needed motor weight based off the required torque.

C. Perforance Evaluation

If one were comparing the performance of different motors that had different torques or speeds, equation 7 allows a more direct comparison of the performance. For example, if one motor had an MNT of 0.08 it would be much higher performance than a motor with an MNT of 0.04.

V. Conclusions

Mass Normalized Torque is a metric that can be useful in comparing performance of different motors for typical industrial applications running under 3600 rpm. If one were to develop a new motor or generator design, this metric would give a way to compare to incumbent technologies. For example, a motor or generator with an MNT greater than 0.1 under 3600 rpm would be a high performance motor.

It is also useful for system tradeoffs when a simple model is required such as in preliminary system design. For example, to compare the system weight for two different gearboxes, Equation 7 gives a first order approximation of how much the motor size would increase with a lower ratio gearbox.

VI. Biographies

Russel H. Marvin graduated from Clarkson University with his BS in 1988 and received his MS from Rensselaer Polytechnic Institute in 1989.

His past employment experience includes Optiwind, MTI, TRC, Plug Power, Axiohm, Kodak, and NCR. Roles have included technical and leadership roles with progressing responsibility leading to his current position as CEO of LC Drives in Potsdam NY. Additionally, he is currently a PhD candidate in Engineering Science at Clarkson University.

Mr. Marvin has specific expertise in many aspects of electro-mechanical systems design and has been working in the clean tech field for the past 18 years. During this time he has been a serial entrepreneur with specific experience in motors, fans, fuel cells, and wind turbines.

Dr. Brian T. Helenbrook's research interests are in the development and application of new numerical simulation techniques for fluid-flows. One aspect of this research is the development of new numerical algorithms that are accurate and efficient for practical engineering problems. Some specific numerical techniques of interest for accomplishing this goal are spectral/hp finite element methods, parallel computing, multigrid methods, and preconditioning/iteration techniques. The second aspect of his research is the application of these algorithms for studying practical engineering problems. Typical problems include the break-up of liquid fuel droplets, spray atomization, flow over boat hulls, wind turbines, and manufacturing processes.

Dr. Helenbrook has a B.S. from University of Notre Dame in 1991 and a Ph.D. from Princeton University in 1997. He is currently Paynter-Krigman Endowed Professor in Engineering Science Simulation at Clarkson University in Potsdam NY.

Dr. Kenneth D. Visser is an Associate Professor in the Department of Mechanical and Aeronautical Engineering and currently the Director of the Center for Sustainable Energy Systems at Clarkson University. He completed his Ph.D. at the University of Notre Dame in 1991. Following a research appointment at NASA Langley he worked at the Boeing Aircraft Company for five years and was involved in development and design aspects of two aircraft: the High Speed Civil Transport and the 767-400ER. Other activities include helping in the design of the America's Cup Team 2000, AmericaOne, and working with Fairchild Dornier Aircraft in Germany. In 2006, Dr. Visser had the opportunity to spend a year at the DLR in Braunschweig, Germany for a year.

Dr. Visser currently teaches senior aircraft design and performance courses at Clarkson University and is the AIAA faculty student advisor. His research interests are primarily experimental, focusing on applied aerodynamics and renewable energy concepts, including wind turbine design optimization, drag reduction of ground vehicles and design methodologies for aircraft wing tips.

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